

Thermodynamics of apparent horizon in modified FRW universe with power-law corrected entropy

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Abstract

We derive the modified Friedmann equation corresponding to the power-law corrected entropy-area relation $S_A = \frac{A}{4} \left[1 - K_\alpha A^{1-\frac{\alpha}{2}} \right]$ which is motivated by the entanglement of quantum fields in and out of the apparent horizon. We consider a non-flat modified FRW universe containing an interacting viscous dark energy with dark matter and radiation. For the selected model, we study the effect of the power-law correction term to the entropy on the dynamics of dark energy. Furthermore, we investigate the validity of the generalized second law (GSL) of gravitational thermodynamics on the apparent horizon and conclude that the GSL is satisfied for $\alpha < 2$.

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1 Introduction

The present acceleration of the universe expansion has been well established through numerous and complementary cosmological observations [1]. One explanation for the cosmic acceleration is the dark energy (DE), an exotic energy with negative pressure. Although the nature and cosmological origin of DE is still enigmatic at present, a great variety of models have been proposed to describe the DE (for review see [2]). One of interesting issues in modern cosmology is the thermodynamical description of the accelerating universe driven by the DE.

In black hole physics, it was found that black holes emit Hawking radiation with a temperature proportional to their surface gravity at the event horizon and they have an entropy which is one quarter of the area of the event horizon [3]. The temperature, entropy and mass of black holes satisfy the first law of thermodynamics [4]. It was shown that the Einstein equation can be derived from the first law of thermodynamics by assuming the proportionality of entropy and the horizon area [5].

The relation between the Einstein equation and the first law of thermodynamics has been generalized to the cosmological context. It was shown that by applying the Clausius relation $-dE = T_A dS_A$ to the apparent horizon \tilde{r}_A , the Friedmann equation in the Einstein gravity can be derived if we take the Hawking temperature $T_A = 1/(2\pi\tilde{r}_A)$ and the entropy $S_A = A/4$ on the apparent horizon, where A is the area of the horizon [6]. The equivalence between the first law of thermodynamics and Friedmann equation was also found for gravity with Gauss-Bonnet term, the Lovelock gravity theory and the braneworld scenarios [6, 7, 8].

Note that in thermodynamics of apparent horizon in the standard Friedmann-Robertson-Walker (FRW) cosmology, the geometric entropy is assumed to be proportional to its horizon area, $S_A = A/4$ [6]. However, this definition for the entropy can be modified from the inclusion of quantum effects. For instance in quantum tunneling formalism, taking into account the quantum back reaction effects in the spacetime found by conformal field theory methods and using the second law of thermodynamics, the corrections to the both semiclassical Bekenstein-Hawking area law ($S_{\text{BH}} = A/4$) and Friedmann equation can be obtained [9]. The quantum corrections provided to the entropy-area relationship lead to the curvature correction in the Einstein-Hilbert action and vice versa [10]. The power-law quantum correction to the horizon entropy motivated by the entanglement of quantum fields between inside and outside of the horizon is given by [11]

$$S_A = \frac{A}{4} \left[1 - K_\alpha A^{1-\frac{\alpha}{2}} \right], \quad (1)$$

where we take $c = k_B = \hbar = G = 1$. Also α is a dimensionless parameter and

$$K_\alpha = \frac{\alpha(4\pi)^{\frac{\alpha}{2}-1}}{(4-\alpha)r_c^{2-\alpha}}, \quad (2)$$

where r_c is the crossover scale. Note that in the case of $\alpha = 0 = K_\alpha$, Eq. (1) reduces to the well-known Bekenstein-Hawking entropy-area relation $S_A = S_{\text{BH}} = A/4$.

Besides the first law of thermodynamics, a lot of attention has been paid to the generalized second law (GSL) of thermodynamics in the accelerating universe driven by

the DE [12, 13, 14, 15, 16, 17, 18]. The GSL of thermodynamics like the first law is an accepted principle in physics. According to the GSL, the entropy of matter inside the horizon plus the entropy of the horizon do not decrease with time [19, 20, 21, 22].

Here, we would like to examine whether the power-law corrected entropy (1) together with the matter field entropy inside the apparent horizon will satisfy the GSL of thermodynamics. To be more general we will consider an interacting viscous DE with dark matter (DM) and radiation. The observations indicate that the universe media is not a perfect fluid and the viscosity is concerned in the evolution of the universe (see [23] and references therein).

This paper is organized as follows. In section 2, using the Clausius relation we derive the modified Friedmann equation corresponding to the power-law corrected entropy (1). In section 3, we study the interacting viscous DE with DM and radiation in a non-flat modified FRW universe. In section 4, we investigate the effect of the power-law correction term to the entropy on the dynamics of DE. Section 5 is devoted to conclusions. In Appendix A, we investigate the validity of the GSL of gravitational thermodynamics with power-law corrected entropy for the universe enclosed by the apparent horizon.

2 Clausius relation and modified Friedmann equation

In the framework of FRW metric,

$$ds^2 = h_{ij}dx^i dx^j + \tilde{r}^2 d\Omega^2, \quad (3)$$

where $\tilde{r}(t) = a(t)r$, $x^i = (t, r)$ and $h_{ij} = \text{diag}(-1, a^2/(1 - kr^2))$, $i, j = 0, 1$, by setting

$$f := h^{ij}\partial_i \tilde{r} \partial_j \tilde{r} = 1 - \left(H^2 + \frac{k}{a^2} \right) \tilde{r}^2 = 0, \quad (4)$$

the location of the apparent horizon in the FRW universe is obtained as [24]

$$\tilde{r}_A = H^{-1}(1 + \Omega_k)^{-1/2}. \quad (5)$$

Here $\Omega_k = k/(a^2 H^2)$ and $k = 0, 1, -1$ represent a flat, closed and open universe, respectively. The Hawking temperature on the apparent horizon is given by [6]

$$T_A = \frac{1}{2\pi\tilde{r}_A} \left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right), \quad (6)$$

where $\frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} < 1$ ensure that the temperature is positive.

To derive the modified Friedmann equation corresponding to the power-law corrected entropy (1) we start with the Clausius relation [5]

$$-dE = T_A dS_A, \quad (7)$$

where $-dE$ is the amount of energy crossing the apparent horizon during the infinitesimal time interval dt in which the radius of the apparent horizon is assumed to be fixed, i.e. $\dot{\tilde{r}}_A = 0$ [25]. This yields $T_A = 1/(2\pi\tilde{r}_A)$.

Following [26] we have

$$-dE = 4\pi\tilde{r}_A^3(\rho + p)Hdt, \quad (8)$$

where ρ and p are the energy density and pressure of the fluid, respectively, inside the universe and satisfy the energy conservation law

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (9)$$

From Eqs. (1), (5), (6) and using $A = 4\pi\tilde{r}_A^2$ one can obtain

$$T_A dS_A = T_A \frac{\partial S_A}{\partial A} dA = - \left(\dot{H} - \frac{k}{a^2} \right) \left\{ 1 - \frac{\alpha}{2} \left[\left(H^2 + \frac{k}{a^2} \right)^{1/2} r_c \right]^{\alpha-2} \right\} \tilde{r}_A^3 H dt. \quad (10)$$

Equating (8) with (10) and using (9) gives

$$2H \left(\dot{H} - \frac{k}{a^2} \right) \left\{ 1 - \frac{\alpha}{2} \left[\left(H^2 + \frac{k}{a^2} \right)^{1/2} r_c \right]^{\alpha-2} \right\} = \frac{8\pi}{3} \dot{\rho}. \quad (11)$$

Integrating with respect to cosmic time t we get the modified Friedmann equation

$$H^2 + \frac{k}{a^2} - r_c^{-2} \left[r_c^\alpha \left(H^2 + \frac{k}{a^2} \right)^{\alpha/2} - 1 \right] = \frac{8\pi}{3} \rho, \quad (12)$$

which in the absence of correction term, i.e. $\alpha = 0$, it recovers the well-known first Friedmann equation in the standard FRW cosmology.

3 Interacting viscous DE, DM and radiation

Here we consider a non-flat FRW universe containing the DE, DM and radiation. Hence the first modified Friedmann equation (12) corresponding to the power-law corrected entropy (1) takes the form

$$H^2 + \frac{k}{a^2} - r_c^{-2} \left[r_c^\alpha \left(H^2 + \frac{k}{a^2} \right)^{\alpha/2} - 1 \right] = \frac{8\pi}{3} (\rho_D + \rho_m + \rho_r), \quad (13)$$

where ρ_D , ρ_m and ρ_r are the energy density of DE, DM and radiation, respectively.

Using the following definitions

$$\Omega_D = \frac{8\pi\rho_D}{3H^2}, \quad \Omega_m = \frac{8\pi\rho_m}{3H^2}, \quad \Omega_r = \frac{8\pi\rho_r}{3H^2}, \quad (14)$$

$$\begin{aligned} \Omega_\alpha &= (Hr_c)^{-2} \left[(Hr_c)^\alpha (1 + \Omega_k)^{\alpha/2} - 1 \right], \\ &= (1 + \Omega_k) \left(\frac{\tilde{r}_A}{r_c} \right)^2 \left[\left(\frac{\tilde{r}_A}{r_c} \right)^{-\alpha} - 1 \right], \end{aligned} \quad (15)$$

one can rewrite Eq. (13) as

$$1 + \Omega_k = \Omega_D + \Omega_m + \Omega_r + \Omega_\alpha. \quad (16)$$

Here, we consider a viscous model of DE. In an isotropic and homogeneous FRW universe, the dissipative effects arise due to the presence of bulk viscosity in cosmic fluids. The DE with bulk viscosity has a peculiar property to cause accelerated expansion of phantom type in the late evolution of the universe [27, 28]. Note that the total energy density still satisfies the conservation law (9) where

$$\rho = \rho_D + \rho_m + \rho_r, \quad (17)$$

$$p = \tilde{p}_D + p_r, \quad (18)$$

and

$$\tilde{p}_D = p_D - 3H\xi, \quad (19)$$

is the effective pressure of the DE and ξ is the bulk viscosity coefficient [23, 28]. Note that $p_r = \rho_r/3$ and the DM is pressureless, i.e. $p_m = 0$. Here like [29], if we assume $\xi = \varepsilon\rho_D H^{-1}$, where ε is a constant parameter, then the total pressure yields

$$p = (\omega_D - 3\varepsilon)\rho_D + \frac{1}{3}\rho_r, \quad (20)$$

where $\omega_D = p_D/\rho_D$ is the equation of state (EoS) parameter of the viscous DE.

We further assume that the viscous DE, DM and radiation interact with each other. Recently the scenario in which the DE interacts with DM and radiation has been introduced to resolve the cosmic triple coincidence problem [30]. In the presence of interaction, the energy conservation laws for the viscous DE, DM and radiation are not separately hold and we have

$$\dot{\rho}_D + 3H\rho_D(1 + \omega_D) = 9H^2\xi - Q, \quad (21)$$

$$\dot{\rho}_m + 3H\rho_m = Q', \quad (22)$$

$$\dot{\rho}_r + 4H\rho_r = Q - Q', \quad (23)$$

where Q and Q' stand for the interaction terms.

Taking a time derivative in both sides of Eq. (13), and using Eqs. (14), (15), (16), (21), (22), (23) and $\xi = \varepsilon\rho_D H^{-1}$, the EoS parameter of interacting viscous DE can be obtained as

$$\omega_D = -\frac{1}{3\Omega_D} \left\{ 2 \left(\frac{\dot{H}}{H^2} - \Omega_k \right) \left[1 - \left(\frac{\alpha}{2} \right) \frac{\Omega_\alpha + (Hr_c)^{-2}}{1 + \Omega_k} \right] + 3\Omega_m + 4\Omega_r \right\} + 3\varepsilon - 1. \quad (24)$$

The deceleration parameter is given by

$$q = - \left(1 + \frac{\dot{H}}{H^2} \right). \quad (25)$$

Substituting the term \dot{H}/H^2 from (24) into (25) yields

$$q = \frac{(1 + \Omega_k)}{2 \left[1 + \Omega_k - \frac{\alpha}{2} (\Omega_\alpha + (Hr_c)^{-2}) \right]} \left[3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r \right] - (1 + \Omega_k). \quad (26)$$

Using Eq. (16) one can rewrite (26) as

$$q = \frac{(1 + \Omega_k)}{2 \left[1 + \Omega_k - \frac{\alpha}{2} (\Omega_\alpha + (Hr_c)^{-2}) \right]} \left[1 + \Omega_k + \Omega_\alpha (\alpha - 3) + \alpha (Hr_c)^{-2} + 3\Omega_D (\omega_D - 3\varepsilon) + \Omega_r \right]. \quad (27)$$

4 The effect of the power-law correction term to the entropy on the dynamics of DE

Here to see how the power-law correction term to the entropy (1) influence the dynamics of DE in our selected model for the universe, we need to incorporate a specific form of the DE model as well as the interaction terms between DE, DM and radiation. To do this we consider the power-law entropy-corrected version of the holographic DE (HDE) model. The HDE model is motivated by the holographic principle [31]. Following [32], the HDE density is given by

$$\rho_D = 3c^2 M_P^2 L^{-2}, \quad (28)$$

where c is a dimensionless constant, M_P is the reduced Planck Mass $M_P^{-2} = 8\pi$ with $G = 1$ and L is the IR cut-off.

Indeed, the definition and derivation of the HDE density depends on the Bekenstein-Hawking entropy-area relation $S_{\text{BH}} = A/4$, where $A \sim L^2$ is the area of horizon. Taking into account the power-law correction (1) to the Bekenstein-Hawking entropy, which appears in dealing with the entanglement of quantum fields between in and out the horizon, the HDE density is modified accordingly. This modification yields the energy density of the so-called “power-law entropy-corrected HDE” (PLECHDE) as [33]

$$\rho_D = 3c^2 M_P^2 L^{-2} - \beta M_P^2 L^{-\alpha}, \quad (29)$$

where β is a dimensional constant. In the special case $\beta = 0$, the above equation yields the well-known HDE density (28).

From definition $\rho_D = 3M_P^2 H^2 \Omega_D$ and using Eq. (29), we get

$$L = \frac{c}{H} \left(\frac{\gamma_c}{\Omega_D} \right)^{1/2}, \quad (30)$$

where

$$\gamma_c = 1 - \frac{\beta}{3c^2} L^{2-\alpha}. \quad (31)$$

If we consider the apparent horizon as an IR cut-off, $L = \tilde{r}_A$, in a non-flat FRW universe, then taking a derivative of Eq. (29) with respect to cosmic time t yields

$$\frac{\dot{\rho}_D}{\rho_D} = \left(\frac{\alpha - 2}{\gamma_c} - \alpha \right) \frac{\dot{\tilde{r}}_A}{\tilde{r}_A}. \quad (32)$$

Taking a time derivative of Eq. (5) gives

$$\dot{\tilde{r}}_A = \frac{\Omega_k - \frac{\dot{H}}{H^2}}{(1 + \Omega_k)^{3/2}}. \quad (33)$$

Taking a time derivative of Eq. (13) and using Eqs. (15), (22) and (23) gives

$$\frac{\dot{H}}{H^2} = \Omega_k + \frac{\frac{8\pi}{3H^3}(\dot{\rho}_D + Q) - 3\Omega_m - 4\Omega_r}{2 - \alpha\left(\frac{\tilde{r}_A}{\tilde{r}_{A0}}\right)^{2-\alpha}}, \quad (34)$$

where following [11, 34] we take $r_c = \tilde{r}_{A0}$. In what follows, following Cruz et al. in [30] we assume

$$Q = 3b^2 H(\rho_D + \rho_m + \rho_r), \quad (35)$$

$$Q' = 3b'^2 H(\rho_D + \rho_m + \rho_r), \quad (36)$$

with the coupling constants b^2 and b'^2 .

Using Eqs. (33), (34) and (35) one can rewrite Eq. (32) as

$$\frac{\dot{\rho}_D}{3H\rho_D} = \frac{\left(\frac{\alpha-2}{\gamma_c} - \alpha\right)\left[\Omega_m + \frac{4}{3}\Omega_r - b^2(1 + \Omega_k - \Omega_\alpha)\right]}{\left(\frac{\alpha-2}{\gamma_c} - \alpha\right)\Omega_D + \left[2 - \alpha\left(\frac{\tilde{r}_A}{\tilde{r}_{A0}}\right)^{2-\alpha}\right](1 + \Omega_k)}. \quad (37)$$

Substituting Eqs. (35) and (37) in (34) gives

$$\frac{\dot{H}}{H^2} = \Omega_k + \frac{3b^2(1 + \Omega_k - \Omega_\alpha) - 3\Omega_m - 4\Omega_r}{\left(\frac{\alpha-2}{\gamma_c} - \alpha\right)\left(\frac{\Omega_D}{1+\Omega_k}\right) + \left[2 - \alpha\left(\frac{\tilde{r}_A}{\tilde{r}_{A0}}\right)^{2-\alpha}\right]}. \quad (38)$$

Inserting Eq. (38) in (24) and using Eq. (15) yields the EoS parameter of the interacting viscous PLECHDE as

$$\begin{aligned} \omega_D = -1 + 3\varepsilon - b^2 \left(\frac{1 + \Omega_k - \Omega_\alpha}{\Omega_D} \right) \\ + \frac{\left(\frac{\alpha-2}{\gamma_c} - \alpha\right)\left[b^2(1 + \Omega_k - \Omega_\alpha) - \Omega_m - \frac{4}{3}\Omega_r\right]}{\left(\frac{\alpha-2}{\gamma_c} - \alpha\right)\Omega_D + \left[2 - \alpha\left(\frac{\tilde{r}_A}{\tilde{r}_{A0}}\right)^{2-\alpha}\right](1 + \Omega_k)}. \end{aligned} \quad (39)$$

Replacing Eq. (38) into (25) gives the deceleration parameter as

$$q = -1 - \Omega_k - \frac{3b^2(1 + \Omega_k - \Omega_\alpha) - 3\Omega_m - 4\Omega_r}{\left(\frac{\alpha-2}{\gamma_c} - \alpha\right)\left(\frac{\Omega_D}{1+\Omega_k}\right) + \left[2 - \alpha\left(\frac{\tilde{r}_A}{\tilde{r}_{A0}}\right)^{2-\alpha}\right]}. \quad (40)$$

In the absence of correction term ($\alpha = 0 = \beta$), from Eqs. (15) and (31) we have $\Omega_\alpha = 0$ and $\gamma_c = 1$, respectively. If we also consider a spatially flat FRW universe ($\Omega_k = 0$), Eq. (5) shows that the apparent horizon is same as the Hubble horizon, i.e. $\tilde{r}_A = H^{-1}$, and $L = \tilde{r}_A = H^{-1}$. Now if we take $\varepsilon = b^2 = \Omega_r = 0$ then Eq. (39) yields the pressureless DE, i.e. $\omega_D = 0$, where its EoS behaves like the dust (or dark) matter. This result has been

already obtained by Hsu [35] for the HDE model with the IR cut-off $L = H^{-1}$. Also from Eq. (40) we obtain $q = 1/2$. Therefore, choosing the Hubble horizon as the IR cut-off $L = H^{-1}$ for the HDE model yields a wrong EoS parameter and cannot drive the universe to accelerated expansion.

Whereas in the presence of the power-law correction term, for the flat FRW universe with $L = \tilde{r}_A = H^{-1}$ from Eq. (30) we have $\gamma_c = \Omega_D/c^2$. Now if we take $\varepsilon = b^2 = \Omega_r = 0$ then Eqs. (39) and (40) for the present time, $\tilde{r}_A = \tilde{r}_{A_0}$, reduce to

$$\omega_{D_0} = -1 - \left(\frac{1}{\Omega_{D_0}} - 1 \right) \left(1 - \frac{1}{1 - c^2 - \left(\frac{\alpha}{2-\alpha} \right) \Omega_{D_0}} \right), \quad (41)$$

$$q_0 = -1 + \frac{3(1 - \Omega_{D_0})}{(2 - \alpha)(1 - c^2) - \alpha \Omega_{D_0}}, \quad (42)$$

where the subscript “0” denotes the present values of the quantities. Taking $\Omega_{D_0} = 0.73$ [1] and $c = 0.818$ [36] then Eqs. (41) and (42) yield

$$\omega_{D_0} = -1.021 + \frac{0.480}{0.624 - \alpha}, \quad (43)$$

$$q_0 = - \left(\frac{\alpha + 0.139}{\alpha - 0.624} \right). \quad (44)$$

The above relations show that for $\alpha > 0.624$ we have $\omega_{D_0} < -1$ and $q_0 < 0$. Here $\omega_{D_0} < -1$ clears that the PLECHDE with the IR cut-off $L = H^{-1}$ behaves like phantom DE. Recent astronomical data indicates that the EoS parameter ω_{D_0} at the present lies in a narrow strip around $\omega_{D_0} = -1$ and is quite consistent with being below this value [37]. On the other hand, the satisfaction of the GSL of gravitational thermodynamics for the universe with the power-law corrected entropy (1) implies that $\alpha < 2$ (see Appendix A). Hence for the PLECHDE model with $0.624 < \alpha < 2$, the identification of IR cut-off with the Hubble horizon $L = H^{-1}$, can derive a phantom accelerating universe which is compatible with the observations.

5 Conclusions

Here, we considered a power-law quantum correction to the entropy of the dynamical apparent horizon motivated by the entanglement of quantum fields between inside and outside of the horizon. Using the Clausius relation we obtained the modified Friedmann equation. For a non-flat modified FRW universe filled with an interacting viscous DE with DM and radiation, we obtained the EoS parameter of interacting viscous DE as well as the deceleration parameter. We studied the effect of the power-law correction term to the entropy on the dynamics of DE in our selected model for the universe. Interestingly enough, we found that for the PLECHDE model which is the power-law entropy-corrected version of the HDE model, the identification of IR cut-off with Hubble horizon, $L = H^{-1}$, can lead to a phantom accelerating universe. This is in contrast to the ordinary HDE where $\omega_D = 0$ if one chooses $L = H^{-1}$. Furthermore, we investigated the validity of the GSL of thermodynamics on the apparent horizon. We found out that the GSL of

thermodynamics with power-law corrected entropy-area relation $S_A = \frac{A}{4} \left[1 - K_\alpha A^{1-\frac{\alpha}{2}} \right]$ is satisfied for $\alpha < 2$.

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A GSL with power-law corrected entropy

Here, we study the validity of the GSL of thermodynamics for the power-law entropy-corrected Friedmann equation. According to the GSL, entropy of the viscous DE, DM and radiation inside the horizon plus the entropy associated with the horizon must not decrease in time.

Taking a time derivative in both sides of Eq. (5), and using Eqs. (9), (13), (14), (15), (16), (17) and (20), one can get

$$\dot{\tilde{r}}_A = \frac{(1 + \Omega_k)^{-1/2}}{2 \left[1 + \Omega_k - \frac{\alpha}{2} (\Omega_\alpha + (Hr_c)^{-2}) \right]} \left[3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r \right]. \quad (45)$$

Using Eq. (26) one can rewrite (45) as

$$\dot{\tilde{r}}_A = \frac{1 + \Omega_k + q}{(1 + \Omega_k)^{3/2}}. \quad (46)$$

From Eqs. (1) and (6), the evolution of the apparent horizon entropy is obtained as

$$T_A \dot{S}_A = 4\pi H \tilde{r}_A^3 (\rho + p) - 2\pi \tilde{r}_A^2 \dot{\tilde{r}}_A (\rho + p). \quad (47)$$

The entropy of the universe including the viscous DE, DM and radiation inside the dynamical apparent horizon can be related to its energy and pressure in the horizon by Gibb's equation [21]

$$TdS = d(\rho V) + p dV = Vd\rho + (\rho + p)dV, \quad (48)$$

where $V = 4\pi \tilde{r}_A^3/3$ is the volume of the universe enclosed by the dynamical apparent horizon \tilde{r}_A . Following [16, 17], we assume that the temperature T of the universe enclosed by the dynamical apparent horizon should be in equilibrium with the Hawking temperature T_A associated with the dynamical apparent horizon, so we have $T = T_A$. Therefore from Eq. (48) one can obtain

$$T_A \dot{S} = 4\pi \tilde{r}_A^2 \dot{\tilde{r}}_A (\rho + p) - 4\pi H \tilde{r}_A^3 (\rho + p), \quad (49)$$

where $S = S_D + S_m + S_r$ is the entropy in the universe containing the viscous DE, DM and radiation. Finally, adding Eqs. (47) and (49), the GSL due to the different contributions of the viscous DE, DM, radiation and dynamical apparent horizon can be obtained as

$$T_A \dot{S}_{\text{tot}} = 2\pi \tilde{r}_A^2 \dot{\tilde{r}}_A (\rho + p), \quad (50)$$

where $S_{\text{tot}} = S + S_A$ is the total entropy.

From Eqs. (17), (20) and using (14) one can obtain

$$\rho + p = \frac{H^2}{8\pi} [3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r]. \quad (51)$$

Using Eq. (26) one can rewrite (51) as

$$\rho + p = \frac{H^2}{4\pi} \left(\frac{1 + \Omega_k + q}{1 + \Omega_k} \right) \left[1 + \Omega_k - \frac{\alpha}{2} (\Omega_\alpha + (Hr_c)^{-2}) \right]. \quad (52)$$

Substituting Eqs. (5), (46) and (52) into (50) yields the GSL as

$$T_A \dot{S}_{\text{tot}} = \frac{(1 + \Omega_k + q)^2}{2(1 + \Omega_k)^{7/2}} \left[1 + \Omega_k - \frac{\alpha}{2} (\Omega_\alpha + (Hr_c)^{-2}) \right], \quad (53)$$

which can be rewritten by the help of Eq. (26) as

$$T_A \dot{S}_{\text{tot}} = \frac{[3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r]^2}{8(1 + \Omega_k)^{3/2} \left[1 + \Omega_k - \frac{\alpha}{2} (\Omega_\alpha + (Hr_c)^{-2}) \right]}. \quad (54)$$

By substituting Ω_α from Eq. (15) in the above equation we get

$$T_A \dot{S}_{\text{tot}} = \frac{[3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r]^2}{8(1 + \Omega_k)^{5/2} \left[1 - \frac{\alpha}{2} \left(\frac{\tilde{r}_A}{r_c} \right)^{2-\alpha} \right]}. \quad (55)$$

According to Eq. (55), the validity of GSL, i.e. $T_A \dot{S}_{\text{tot}} > 0$, depends on the sign of the expression $\left[1 - \frac{\alpha}{2} \left(\frac{\tilde{r}_A}{r_c} \right)^{2-\alpha} \right]$ appearing in the denominator. Hence the GSL is hold when

$$1 - \frac{\alpha}{2} \left(\frac{\tilde{r}_A}{r_c} \right)^{2-\alpha} = 1 - \frac{\alpha}{2} \left(\frac{\tilde{r}_A}{\tilde{r}_{A_0}} \right)^{2-\alpha} > 0, \quad (56)$$

where following [11, 34] we identify the crossover scale r_c with the present value of the apparent horizon \tilde{r}_{A_0} . Since $\tilde{r}_A/\tilde{r}_{A_0}$ tends to zero in the far past and its today value is $\tilde{r}_A/\tilde{r}_{A_0} = 1$, hence for $0 \leq \tilde{r}_A/\tilde{r}_{A_0} \leq 1$ the condition (56) is satisfied when $\alpha < 2$. Therefore the GSL with the power-law corrected entropy (1) is respected for $\alpha < 2$.

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